NEUTRON STARS

Hartle § 13.1, 11.2+3, 24

- Do we need GR? Check compactness

$$\frac{R_S}{R_{NS}} = \frac{\frac{2GM_{NS}}{c^2}}{R_{NS}} = \frac{\underbrace{\frac{2GM_\odot}{c^2}}_{3\,km} \cdot 1.4}{10\,km} \sim 0.5 \Rightarrow \text{definitely GR}$$

- Hydrostatic equilibrium of relativistic stars

sph. symmetry: $$ds^2 = -e^{\nu(r)}dt^2 + \underbrace{e^{\lambda(r)}}_{=\frac{1}{1-\frac{2m(r)}{r}}} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

r = "areal radius"

metric into Einstein Eq. w/ ideal fluid at rest:

$$\rightarrow \quad \frac{dm}{dr} = 4\pi r^2 \rho(r) \qquad (1)$$

$$-\frac{dp}{dr} = [\rho(r) + p(r)] \frac{m(r) + 4\pi r^3 p(r)}{r^2\left(1 - \frac{2m(r)}{r}\right)} \qquad (2)$$

TOV Equations

$$\frac{1}{2}\frac{d\nu}{dr} = \frac{m(r) + 4\pi r^3 p(r)}{r^2\left(1-\frac{2m(r)}{r}\right)} \qquad (3)$$

Tolman
Oppenheimer
Volkov

TOV notes

1) $p(r)$, $\rho(r)$ linked by EOS: $p = p(\rho)$

2) $\rho = \rho_0 (1 + \varepsilon)$ ← internal energy per rest mass

ρ: total energy density

ρ_0: rest-mass density

3) (1) $\Rightarrow$ $m(r)$ = mass inside r

4) To solve:

   i) choose central density $\rho_c = \rho(0)$

   ii) integrate (1), (2) outward until $p = 0$ (surface)

   iii) at surface and outside of star:

   Schwarzschild spacetime $\Rightarrow$ $m(R) \equiv M$ mass of star

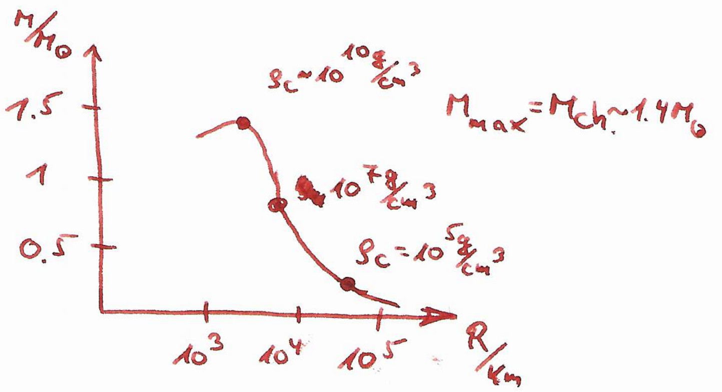
   $e^{\nu(R)} = 1 - \frac{2M}{R}$

   iv) integrate (3) inward

   v) repeat for many ρ_c

• Results of TOV:

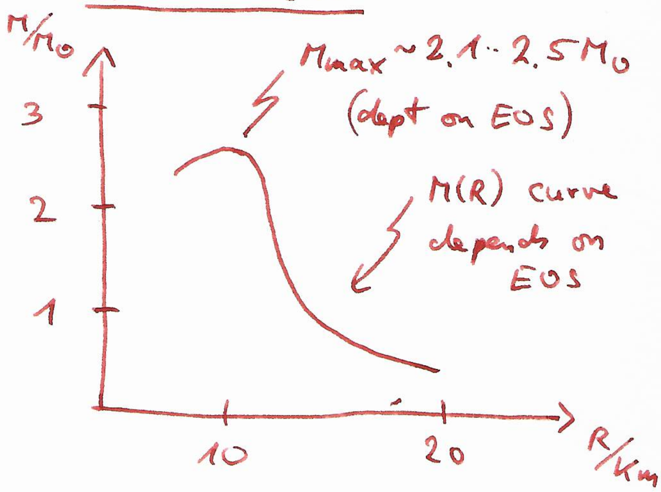
White dwarf



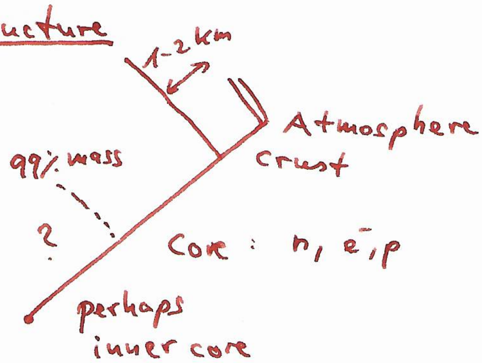
configurations with $\frac{dM}{dR} < 0$ unstable

$\Rightarrow$ collapse to NS

Neutron star

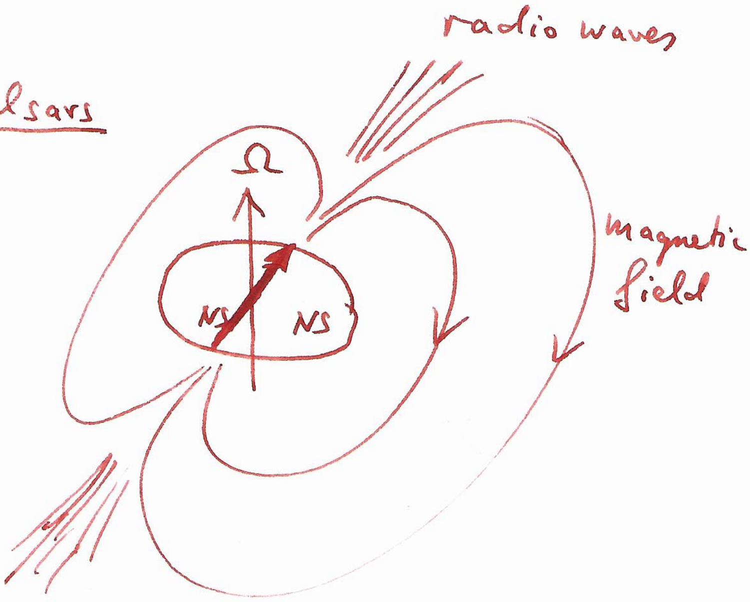


NS structure



w/ exotic matter phases
hyperions, quarks, ...

Pulsars



rotating NS
with misaligned
magnetic dipole field.

radio beam hits
Earth once per rotation

discovered by Jocelyn Bell 1967
several 1000's in MW known
$P \sim 10\,\text{sec} \ldots 1.2\,\text{ms}$ (!)

extremely stable
→ pulsar timing

- Bound on density

$P = \frac{2\pi}{\Omega}$ rotation period

$$\Omega^2 R < \frac{GM}{R^2} \qquad \bar{\varrho} = \frac{M}{\frac{4\pi}{3}R^3}$$

$$\left(\frac{2\pi}{P}\right)^2 < \frac{GM}{R^3} = \frac{4\pi}{3} G \bar{\varrho}$$

$$\bar{\varrho} > \frac{3\pi}{GP^2} \stackrel{\uparrow}{=} 10^{11} \frac{g}{cm^3} \quad !$$

Crab $P = 33\,ms$

- Rotational energy loss

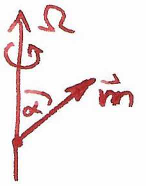
$$E_{rot} = \frac{1}{2} I \Omega^2 = \frac{2\pi^2 I}{P^2}$$

I = moment of inertia $\sim MR^2 \sim 10^{45}\,g\,cm^2$

$$\dot{E}_{rot} = -\frac{4\pi^2 I}{P^3}\dot{P} \stackrel{\uparrow}{\simeq} 10^5 L_\odot \quad !$$

Crab $\dot{P} = 10^{-12.4}$

- Magnetic dipole radiation



magnetic moment $m = BR^3$

$$P_{rad} = \frac{2}{3}\frac{(m \sin\alpha)^2 \Omega^4}{c^3} = \frac{2(2\pi)^4}{3c^3}(BR^3 \sin\alpha)^2 P^{-4}$$

$$P_{rad} \stackrel{!}{=} -\dot{E}_{rot}$$

$$\Rightarrow B^2 = \underbrace{\frac{3c^2 I}{8\pi^2 R^6}}_{\sim (3\times10^{19}G)^2/s} \underbrace{\frac{1}{\sin^2\alpha}}_{\geq 1} P\dot{P} \qquad (*)$$

$$B \gtrsim 3\times10^{19}G\sqrt{\frac{P\dot{P}}{s}} \stackrel{\uparrow}{\simeq} 4\times10^{12}G \quad !$$

Crab

- characteristic age

  assuming $B = \text{const}$, $(*) \Rightarrow$

  $$P\dot{P} = \text{const} \qquad (**)$$

$$\Rightarrow \int_0^\tau dt \Rightarrow \frac{1}{2}\left(P(\tau)^2 - P_0^2\right) = \text{const}\ \tau$$

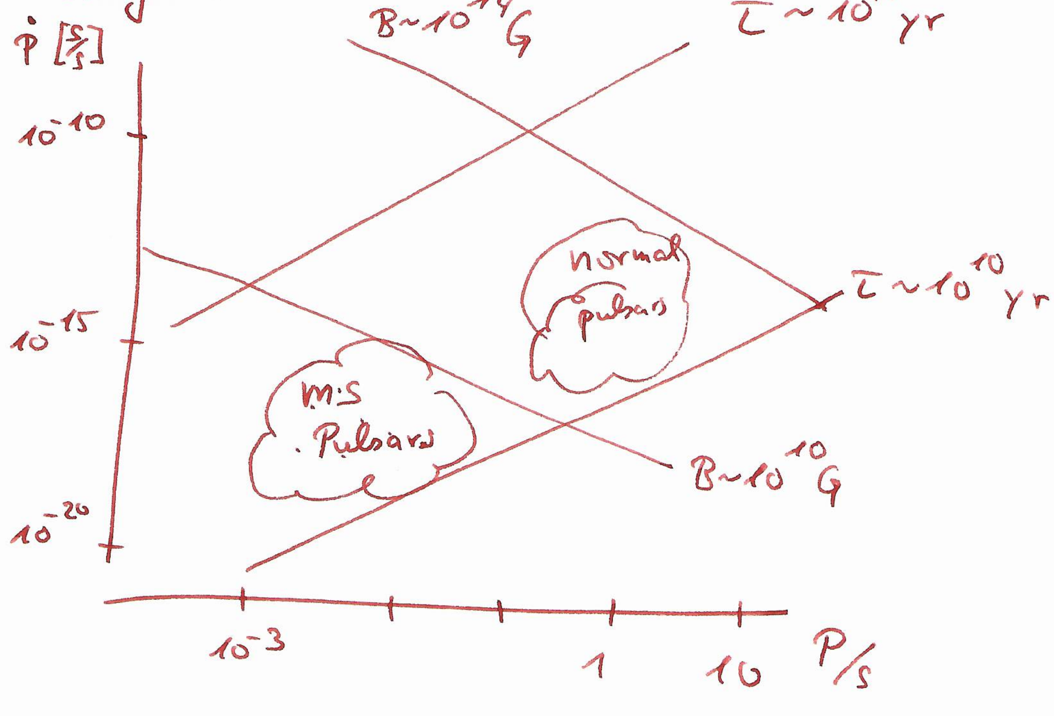
rotation period at birth

age

if $P_0 \ll P(\tau)$: $\tau = \frac{P(\tau)^2}{2\,\text{const}} \overset{(**)}{=} \frac{P}{2\dot{P}}$ "characteristic age"

Crab: $\tau \sim 1300\,\text{yr}$. cf supernova 1054 AD

- $P\dot{P}$ diagram



Binary Pulsars

- two NS orbiting, ≥ 1 as pulsar
  pulsar timing allows to reconstruct orbit with astonishing precision
  (incl. various GR effects, see slides)

Hulse-Taylor Pulsar: B 1913+16

$P \sim 8\,\mathrm{hr}$

$e \sim 0.6$

$1.44 + 1.39\, M_\odot$

$t_{GW} \sim 10^8\, \mathrm{yr}$

J 0737-3039

$P \sim 2.5\,\mathrm{h}$

$\iota \sim 89^\circ$ nearly edge on!

<u>both</u> NS are pulsars