Complex Analysis

Emmanuel Kowalski

1. 0920 ntroduction to the course, examples of applications, definition of holomorphic functions, algebraic stability properties of holomorphic functions.
2. 0921 Convergent power series are holomorphic. Examples and counterexample (the complex conjugate function).
3. 0927 Holomorphy and differentiability; the Cauchy-Riemann equations. Line integrals.
4. 0928 Line integrals and primitives.
5. 1004 Chapter 3: Cauchy's Theorem. Goursat's Theorem, existence of primitives in a circle, Cauchy's Integral Formula.
6. 1005 Chapter 3: proof of Goursat's Theorem.
7. 1011 Chapter 4: applications of Cauchy's Theorem and integral formula: analyticity, Cauchy's inequalities for derivaties, Liouville's Theorem.
8. 1012 Chapter 4: zeros of holomorphic functions, analytic continuation.
9. 1018 Chapter 4: proof of the principle of analytic continuation. Limits of holomorphic functions, Morera's theorem.
10. 1019 Chapter 4: holomorphic functions defined by integrals
11. 1025 Chapter 5: singularities and meromorphic functions, residue theorem.
12. 1026 Chapter 5: residue theorem and examples.
13. 1101 Chapter 5: meromorphic functions, counting zeros, open image and maximum modulus principle.
14. 1102 Chapter 5: meromorphic functions, counting zeros, open image and maximum modulus principle.
15. 1115 Chapter 6: Eta, THeta, Zeta (a long example). Definitions of the functions, infinite products..
16. 1116 Chapter 6: Eta, THeta, Zeta. Analytic continuation of the zeta function, application to prime numbers.

17. 1121 Chapter 6: Eta, THeta, Zeta (a long example). Sketch of Riemann's approach to counting primes; the Riemann Hypothesis.
18. 1123 Chapter 7: Homotopy and applications. Definition and statement of Cauchy's Theorem for homotopic curves.
19. 1129 Chapter 7: Proof of Cauchy's Theorem for homotopic curves.
20. 1130 Chapter 7: simply-connected open sets, existence of primitives. The complex logarithm.
21. 1206 Chapter 7: The residue theorem and homotopy; winding numbers.
22. 1207 Chapter 8: conformal mapping (definition, first examples).
23. 1213 Chapter 8 (conformal mapping): more examples, statement of Riemann's mapping theorem. Outline of the proof. Schwarz Lemma, automorphisms of the disc.
24. 1214 Chapter 8 (conformal mapping); reduction of Riemann's Theorem to the existence of an extremum.
25. 1220 Chapter 8 (conformal mapping): end of the proof of Riemann's Theorem; Montel's Theorem. Final remarks.
26. 1221 Review of the course, questions

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