Analysis

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Analyis I

1. Real Numbers and the completeness
2. Limits and continuity in metric space
3. Open and closed sets in metric space
4. Topological space & The closure of the set
5. Equivalence of continuous functions & Inheritance Principle & Homeomorphism
6. Product metric spaces & Completeness & Compactness (sequentially compact)
7. Compactness (2) & Bolzano-Weierstrass Thm & Nested seq of nonempty compact sets
8. ontinuous functions and Homeomorphisms on compact sets & Connectedness
9. Some properties in topological spaces
10. Accumulation points and isolated points & Dense & Hausdorff space
11. Properties of Connectedness
12. Path connected & Compactness in metric space, Hausdorff space, Topological space(1)
13. Compactness (2) & Sequentially compact implies compact(open cover) (1)
14. Sequentially compact implies compact(open cover) (2) & Totally bounded (1)
15. Totally bounded (2) & Perfect & Cantor set
16. Cantor surjection Theorem
17. Peano Curves & Riemann integrable & Darboux integrable(1)
18. Darboux integrable(2) & Riemann integrable iff Darboux integrable(1)
19. Riemann integrable iff Darboux integrable(2) & Some integrable functions (1)
20. Some integrable functions (2) & Lebesgue number & Zero set
21. Oscillation & Riemann-Lebesgue Theorem
22. Some applications of R-L theorem

23. Fundamental Theorem of Calculus & Convergence in functional space
24. Complete functional space & Convergence function series & Integrals wst unif conv.
25. Derivatives w.s.t uniform convergence & Equicontinuity and Arzela-Ascoli Theorem
26. Heine-Borel Theorem in a function space & Stone-Weierstrass Theorem
27. Fixed point, contraction and ODE
28. Nowhere differentiable continuous function & Baire' s Theorem
29. Corollary of Baire' s Theorem
30. Multiple integral and Fubini' s Theorem
31. Change of variables formula (1)
32. Change of variables formula (2)
33. Banach spaces
34. Banach algebra
35. Differentiation in Banach Spaces
36. Some examples & Integration in Banach spaces (1)
37. Integration in Banach spaces (2)
38. Inverse function theorem
39. Lebesgue Outer measure
40. Lebesgue measurable set & Abstract outer measure
41. Measurable Sets form Sigma-Algebra & Measure continuity
42. Regularity (1)
43. Regularity (2)
44. Every open set is a countable disjoint union of balls plus a zero set
45. Measure Product & Inner measure (1)
46. Inner measure (2) & Slice Measure (1)
47. Slice Measure (2) & Lebesgue integrable functions & Monotone Convergence Theorem

48. Completed undergraph & Dominated Convergence Thm and Fatou's Lem & Basic properties
49. Some basic properties of measurable functions (2)
50. Cavalieri's principle & Equivalence measurable function definition & Tonelli Thm
51. Vitali covering Lemma
52. Density Theorem

Analyis II

1. Lebesgue Differentiation Theorem (1)
2. Lebesgue Differentiation Theorem (2) & Absolutely continuous functions
3. Indefinite integral is abs conti. & A monotone function has a derivative a.e. (1)
4. A monotone function has a derivative almost everywhere (2)
5. Riemann integrals vs Lebesgue integrals
6. Bounded variation
7. Equi relation of B-V functions and abs conti functions & R-S integrals (1)
8. Riemann-Stieltjes integral (2)
9. Riemann-Stieltjes integral (3)
10. Riemann-Stieltjes integral (4)
11. L^2 $[a, b]$ space
12. Properties of L^2 $[a, b]$ space
13. Orthogonal basis in L^2
14. Fourier coefficients and Parseval's formula
15. A necessary condition of Fourier series converges in L^1 & Riemann-Lebesgue lemma
16. Dirichlet kernel
17. Dini Theorem & There is a continuous function whose Fourier series diverges (1)
18. There is a continuous function whose Fourier series diverges (2) & U-B Principle

19. There is a continuous function whose Fourier series diverges (3)

20. Some properties of Fourier coefficients & Cesàro summability

21. Fejér's Theorem & Approximate integrable functions by tri-polynomial in L^1 norm

22. The average of Fourier series converges in L^p norm

23. Equi. of the average of Fourier series converges unif & Weakly convergence(1)

24. Weakly convergent(2) & Equi condition for tri-series to be the Fourier series

25. Convergence of Cesàro mean of Fourier series (1)

26. Part2 Conv of Cesàro mean of Fourier series(2), Conjugate Fourier series & Dini theorem

27. Singular integrals and conjugate function

28. Bound of the L^2 norm of conjugate function & Hardy-Littlewood Maximal function

29. Bound of the norm of the supremum of conjugate functions

30. A counterexample & C-Z Decomposition & The conjugate function exists a.e. in L^1

31. The conjugate function exists a.e. in L^1 (2)

32. The conjugate function exists a.e. in L^1 (3) & Weakly estimate

33. Applications of Fourier series -isoperimetric inequality

34. Applications of Fourier series -continuous but nowhere differentiable function

35. Fourier transform and some properties

36. Fourier inversion formula and Minkowski's inequality

37. Poisson summation formula & Roth's theorem on 3-term AP

38. Discrete Fourier series, Fourier transform and some properties (1)

39. Discrete Fourier series, Fourier transform and some properties (2)

40. Bourgain's theorem

41. Proof of Roth's theorem on 3-terms AP (step 1)

42. Proof of Roth's theorem on 3-terms AP (step 2 to step 7)

43. Proof of Roth's theorem on 3-terms AP (step 9 , step 8)

44. Proof of Roth's theorem on 3-terms AP (step 8)

45. There is f in L^1 such that for a.e. x, the Fourier series diverges (1)

46. There is f in L^1 such that for a.e. x, the Fourier series diverges (2)

47. There is f in L^1 such that for a.e. x, the Fourier series diverges (3)

48. There is f in L^1 such that for a.e. x, the Fourier series diverges (4)

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