Functional Analysis

Lectures by Claudio Landim

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Introduction

These lectures are mainly based on the book Functional Analysis by Peter Lax. Lessons 33 to 37 follow Chapter 4 of the book Applied Functional Analysis by Eberhard Zeidler, volume 108 of Springer's collection Applied Mathematical Sciences.

There are many other very good references. For instance, the books by Bachman and Narici, Brezis, Reed and Simon, Yosida.

These notes were live-TeXed, though I edited for typos and added diagrams requiring the TikZ package separately. I used the editor TeXstudio.

I am responsible for all faults in this document, mathematical or otherwise; any merits of the material here should be credited to the lecturer, not to me.

Please email any corrections or suggestions to jaafar_zhang@163.com.

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I would like to especially thank the IMPA and Professor Landim who put their courses in website.

Lecture 1

Linear Space Definition, Examples and Linear Span

Definition 1.1. X is a linear space over a field $\mathbb{K} \to \mathbb{R}$ *or* $\mathbb{C}$ if X is a set 2 operations $+$ and $\cdot$ multiply by scales

Lecture 2

Linear Spaces: Quotient Spaces and Convex Sets

Lecture 3

Normed Linear Spaces: Definition and Basic Properties

Lecture 4

Completing a Normed Linear Space

Lecture 5

Finite Dimensional Linear Spaces

Lecture 6

Examples of Normed Linear Spaces

Lecture 7

In Infinite Dimensions the Unit Ball is NOT Compact

Lecture 8

Zorn's Lemma

Lecture 9

The Hahn-Banach Theorem

Lecture 10

Convex Sets and Gauge Functions

Lecture 11

Geometric Hanh-Banach Theorems

Lecture 12

Dual of a Normed Linear Space

$X, L.S.$ $\mathbb{K}$ $l : X \to \mathbb{K}$ is a linear function, $\forall x, y \in X, l(x+y) = l(x) + l(y)$, $\forall \alpha \in K,\ l(\alpha x) = \alpha l(x)$

Definition 12.1. l L.F. is continuous if

$$\forall x_n \to x \ \Rightarrow \ l(x_n) \to l(x) \tag{12.1}$$

Definition 12.2. content...

Proposition 12.1. *content...*

Lecture 13

Extension of Bounded Linear Functionals Closed Linear Spans

Lecture 14

Reflexive Spaces

Lecture 15

Hilbert Spaces

Lecture 16

Closed Convex Subsets of a Hilbert Space

Lecture 17

Riesz and Lax-Milgram Representation Theorems

Lecture 18

Riesz and Lax-Milgram Representation Theorems

Lecture 19

Orthonormal Bases

Lecture 20

Uniform Boundedness Principle

Lecture 21

Weak Convergence

Lecture 22

Uniform Boundedness of Weak Converging Sequences

Lecture 23

Weak Sequentially Compactness

Lecture 24

$Weak^*$ Topology

Lecture 25

Applications of Weak Convergence

Lecture 26

Bounded Linear Operators

Lecture 27

Transpose of Bounded Linear Operators

Lecture 28

Strong and Weak Convergence of Operators

Lecture 29

Principle of Uniform Boundedness for Maps and Compositions

Lecture 30

Open Map Principle

Lecture 31

The Closed Graph Theorem

Lecture 32

Examples of Bounded Linear Maps: Integral Operators

Lecture 33

Symmetric Operators

Lecture 34

Eigenvalues of Compact Symmetric Operators

Lecture 35

The Fredholm Alternatives

Lecture 36

An Application to Integral Operators